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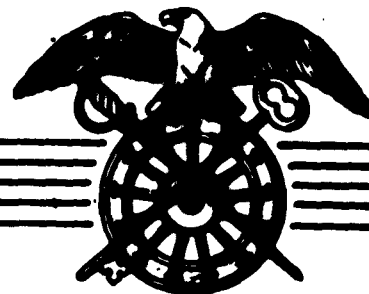
**REVIEW  
OF THE  
HARBRIDGE HOUSE TREATMENT  
OF  
INVENTORY POLICY  
AT THE NATIONAL LEVEL**

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**DEPARTMENT OF THE ARMY  
OFFICE OF THE QUARTERMASTER GENERAL**

**REVIEW OF THE HARBRIDGE HOUSE TREATMENT OF  
INVENTORY POLICY AT THE NATIONAL LEVEL**

**by**

**Lt. Andrew C. Stedry**

**November 1961**

**QM OPERATIONAL MATHEMATICS SERIES  
REPORT NO. 7**

## FOREWORD

Much has been said and done recently in connection with the study by Harbridge House of economic inventory policies (EIP). Since the principle of balancing holding cost against order cost so as to minimize total cost makes good sense, these policies are being implemented by the Army at its posts, camps and stations.

The next logical step then is the application of the results of the study to establishing supply policies at the national level -- specifically at the National Inventory Control Points (NICP's). However, as with any study based on a mathematical model, the assumptions and analyses must be subjected to careful technical scrutiny before the study is given its acid practical test in pilot supply applications. This technical scrutiny was carried out by Lt. A. C. Stedry, a young Ph.D., in Management Science with a strong mathematical background, who was assigned to the Quartermaster Corps' Operational Mathematics Office. The results of his review are reported in this, the seventh in the series of Operational Mathematics Reports.



B. E. KENDALL

Brigadier General, USA  
Acting Deputy The Quartermaster General

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## ABSTRACT

As a part of their extensive contractual effort in the area of economic inventory policies (EIP), Harbridge House of Boston, Massachusetts, studied inventory policies at the national or whole-sale level. Their work was reviewed by the Quartermaster Corps' Operational Mathematics Office. It was found that though the determination of operating levels is adequate, their approach to the determination of safety levels has several flaws making their recommended determination of safety levels inadequate. Mathematical arguments are given for this criticism. Since acceptable alternative approaches are available, it is suggested that these be used to determine national inventory policies.

## INTRODUCTION

As a part of their extensive contractual effort in the area of economic inventory policies (EIP), Harbridge House of Boston, Massachusetts, studied both the wholesale or national level policies and retail policies. The Operational Mathematics Office has assisted in the implementation of the retail policies. An Operational Mathematics Series Report on this assistance is now being prepared. It is sufficient for purposes of the present report to say that it is generally agreed that the Harbridge House treatment of retail accounts would be superior to the Army Field Stock Control approach and that experience to date at posts, camps and stations converted to the Harbridge House approach bears this out.

The national level policies are another question, however. Here, flaws in the approach that are minor in terms of application to retail accounts may not remain minor at all. Also other work on this subject is available, and it is important to review the Harbridge House approach at this time so that a decision can be made on what procedures would be best implemented. This was done. The results of the review are presented in this report.

The remainder of this report is divided into three major sections. In the first, a complete summary of the conclusions reached in the review is given. In the second, the mathematical arguments are developed that led to the chief criticisms of the Harbridge House approach. In the third, the estimate of supply effectiveness is discussed in mathematical terms.

A remark about the footnotes and formulas. Superscript numbers inclosed in brackets will be used to indicate footnotes. References are given in these footnotes. Formulas appearing in the mathematical discussion will be indexed by numbers inclosed in parentheses. These numbers will generally appear to the left of the formula.

## SUMMARY OF THE REVIEW

The Harbridge House approach has flaws which while not crucial at installation level, would become crucial if a direct extrapolation to the national level of a system devised on similar principles were made. More specifically, this extrapolation would result in gross over-protection on some items (i.e., "dead stock") and wholly inadequate protection (i.e., poor supply performance) on others.

The determination of operating levels utilized by Harbridge House is derived directly from the standard "optimum-lot-size" formula which has been known for about 45 years. While it rests on fairly restrictive assumptions -- fixed cost to reorder independent of procurement value, holding cost proportional to dollar value of half the operating level -- it has been found to give results that are close to optimal for less restrictive assumptions. As refinements are extremely complex and their incremental gain is small compared to the gain that can be achieved through improvements in safety level procedure, they will be ignored here.

The determination of safety levels in the subject procedure rests on much less firm ground. The authors of EIP indicate a preference for a gamma distribution to describe demand patterns. They have not attempted to determine whether this distribution in fact describes the demand patterns reasonably well (or at least do not report such an attempt except to show that a few chosen items happen to look more "gamma" than "Poisson"). Furthermore, the method used does not in fact measure the variability of demand but assumes the demand variability to be related to the average order size. As they give no proof that this relationship is an appropriate one, it is quite likely that the safety levels chosen will either give too much protection or too little -- perhaps on most items in the same direction, perhaps not. In any case, it is unlikely that the predicted probabilities will be experienced in practice. The next major section of this report, entitled "Safety Levels", contains a mathematical summary of the Harbridge House procedures and the mathematical arguments that led us to conclude that the safety level procedures developed by Harbridge House are inadequate.

A second major objection to the EIP methodology is its failure to take into account the effect of procurement cycle on supply effectiveness for a given safety level. If the assumptions about demand distributions were correct, the safety levels computed according to the

procedures would provide a 10% probability of stockout during lead time<sup>[1]</sup> -- i.e., during ten periods following the placement of a procurement action, a stockout would be expected in one of them. But each time an item is procured, we are subject to stockout. Therefore, an item ordered monthly will be out of stock six times as often as an item which is ordered once every six months if a procedure is followed which gives the same protection against stockout "across the board". I.e., if an item would have a 95% supply effectiveness if ordered twice a year, it would then have only a 70% supply effectiveness if ordered monthly.

It should be pointed out that a 90% protection against stockout does not mean 90% supply effectiveness. Using the Harbridge House assumptions about distribution, it is possible to determine the expected (average) number of demands which will be placed during a lead time against zero stock for an item given the average number of demands it will experience during a lead time and the probability of stockout. The number of demands against zero stock expected per year would then be equal to the expected number of demands during lead time times the annual order frequency (the latter is the "operating level factor" in Harbridge House parlance). Supply effectiveness can then be expressed as follows:

% Supply effectiveness

$$= 100 \left[ 1 - \frac{\text{Expected number of demands against zero stock per year}}{\text{Expected number of demands per year}} \right]$$

The supply effectiveness resulting from the procedures recommended by Harbridge House, if their assumptions about demand distribution are correct, is shown in Table 1. A mathematical treatment of the method used to relate protection against stockout to supply effectiveness is shown in the last portion of this report.

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[1] For procurement cycles of six months or less. For "annual buy" items no safety level but a 30-day additional "order and ship time" is allowed or, in effect, a 30-day safety level. No probability of stockout is estimated.

TABLE 1. Per cent Supply Effectiveness for Selected Annual Demand and Ordering Frequencies (Utilizing Harbridge House Assumptions and Procedures for a 30-Day Lead Time)

Operating Level Factor*	Annual Demand Frequency†			
	6	12	24	72
1**	97.9	98.9	99.5	100.0‡
2	97.2	98.4	99.1	99.4
4	94.4	96.8	98.2	98.8
6	91.6	95.1	97.3	98.2
12	83.2	90.3	94.5	96.4

\* This term is used to refer to the annual ordering frequency. An item which the agency procures monthly will have an Operating Level factor of 12, annually 1, etc.

\*\* The "annual buy" safety level is, in effect, 30 days' supply. All others are computed on the basis of 90% probability of no stockout during a lead time.

† The average number of requisitions placed on the inventory control point per year.

‡ To 1 decimal-place accuracy.

The effect of demand frequency and ordering frequency on supply effectiveness is readily seen from Table 1. Noting that the Harbridge House assumptions probably underestimate variance, these supply effectiveness percentages must be viewed as being maxima rather than actual.

Use of a more suitable distribution would undoubtedly lower the whole set, attenuate the effect of increasing demand frequency on improved performance but markedly increase the deteriorating effect of increasing procurement frequency. In any case there is apparently no rationale provided for using a scheme which produces this markedly differing performance.

It is believed that application of a scheme similar to the station level scheme which uses (probably) the wrong distribution and an irrational determination of safety levels would be very costly. It is therefore recommended that, at national control points a more sophisticated scheme be used. Such a scheme is contained in the series of reports by the Operations Research Group at MIT<sup>[2]</sup> or in unpublished work done by the Operational Mathematics Office for the Military General Supply Agency. Either of these approaches to national inventory policies would be adequate and are entirely compatible with the established Harbridge House procedures in the posts, camps and stations.

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<sup>[2]</sup> Interim Technical Reports Nos. 7 and 9, MIT Project: Fundamental Investigations in the Methods of Operations Research under contract DA-19-020-ORD-2684, Ordnance R&D Project No. TB-001, August 1958.

## SAFETY LEVELS

### I. Introduction.

The point to be made in this portion of the report is that the safety levels presented by Harbridge House, Inc.,<sup>[3]</sup> for economical inventory policies are not developed from the proper use of the gamma distribution as claimed. We agree with Harbridge House on the applicability of the gamma distribution. In fact, this very agreement is the basis of our objections since, by their treatment of the distribution, Harbridge House analysts have transformed the distribution into something else, as will be explained below.

It will be necessary from time to time to introduce some mathematical formulae. It is our belief that the non-mathematician may ignore these symbols while the technician will be able to understand this section without having to go through the algebraic gymnastics himself. Since the authors of the Harbridge House reports have avoided writing down their assumptions in mathematical form, the technician cannot render his opinion as to the validity of their analysis in an hour or two. Instead, he must spend an immense amount of time "finding out what they did" before he can test its validity. This time constraint may prove prohibitive, in which case the report may pass through many hands, receive many signatures and even be implemented, before receiving extensive technical scrutiny. The material discussed in this portion of the report is a case in point.

### II. The Gamma Distribution.

First, it is desirable to show, for the reader who has some mathematical training the meaning of this distribution. If  $x$  is a gamma-distributed variable, then the frequency distribution of  $x$  may be expressed by:

---

[3] See in particular, "Field Test of the Economic Inventory Policy Report No. 1, The Implementation Phase", Appendix II. The reader may also be interested in seeing Reports Nos. 2, 4, 3, 5 in this series by Harbridge House.

$$(1) \quad r(x) = \frac{e^{-x/\beta} x^p}{\beta^{p+1} p!}$$

where  $p$  and  $\beta$  are "parameters" of the distribution. These parameters, taken together, tell us two things about  $x$ : its mean (or average) which shall be called  $\mu$  and its variance, which shall be called  $\sigma^2$ . Specifically, as may be found in standard statistical texts:

$$(2) \quad \begin{aligned} \mu &= \beta(p + 1) \\ \sigma^2 &= \beta^2(p + 1) \end{aligned}$$

It should be clear to the non-mathematical observer as well as the mathematician that one must know (or be able to estimate) both the mean and the variance<sup>[4]</sup> in order to determine  $\beta$  and  $p$ . Solving for  $p$  and  $\beta$  we obtain:

$$(3) \quad \begin{aligned} \beta &= \sigma^2/\mu \\ p &= \mu/\beta - 1 = \frac{\mu^2 - \sigma^2}{\sigma^2} \end{aligned}$$

again emphasizing the need to know two characteristics of the distribution rather than one in order to define it. However, it will be noted in the Harbridge House reports already referred to that the plan presented requires only the estimate of an average -- not a variance. This, in itself is sufficient to show that the plan used by Harbridge House does not take demand variability into account but only average demand and, as will be evident below, average demand size.

---

<sup>[4]</sup> or the standard deviation -- the square root of the variance.

The process by which the two parameter gamma distribution is reduced to a one-parameter distribution may appear complex to the layman, but it is transparent to the mathematician.

If we call  $t$  the number of items demanded in a delivery time, then the probability that  $t$  will be less than or equal to some value  $x$  is expressed by the cumulative distribution:

$$(4) \quad F(x) = \int_0^x \frac{e^{-t/\beta}}{\beta^{p+1}} \frac{t^p}{p!} dt.$$

A substitution can be made that renders this formula more convenient for computation. Let  $y = t/\beta$ . Then:

$$(5) \quad F(x) = \int_0^{x/\beta} \frac{e^{-y}}{p!} y^p dy.$$

Since:

$$(6) \quad \int_0^\infty e^{-y} y^p dy = p!$$

the expression for  $F(x)$  becomes.<sup>[5]</sup>

<sup>[5]</sup> The tables used for computing values of the cumulative gamma distribution are Karl Pearson (ed.), Tables of the Incomplete  $\Gamma$  Function, London: His Majesty's Stationery Office, 1922. These tables make one further substitution, namely:

$$u = x/\sigma$$

Then

$$\frac{x}{\beta} = \frac{x \sqrt{p+1}}{\sigma} = u \sqrt{p+1}$$

and the function may be written:

$$I(u, p) = \frac{\int_0^{u \sqrt{p+1}} e^{-y} y^p dy}{\int_0^\infty e^{-y} y^p dy}$$

It remains, nevertheless, the probability that the number of items demanded will be less than or equal to  $x$ .

$$(7) \quad F(x) = \frac{\int_0^{x/\beta} e^{-y} y^p dy}{\int_0^{\infty} e^{-y} y^p dy}$$

To clear up any confusion that may exist on this point, the denominator of the above expression, since the function  $e^{-y} y^p$  is integrated over the entire range from zero to infinity is known as the "complete gamma function", usually denoted by  $\Gamma(p+1)$  or more simply  $\Gamma(p+1)$ . In the numerator, the function is integrated over part of the range (from zero to  $x/\beta$ ) and in this case is denoted by  $\Gamma_{x/\beta}(p+1)$ . It should be emphasized that there is nothing "unfinished" about the incomplete gamma function.

### III. The Crucial Assumption.

Although the substitution  $y = t/\beta$  in equation (5) is a valid one, Harbridge House analysts have carried this substitution into a transformation of the distribution. They have defined  $B$  as the average demand size and then assumed, without any justification that

$$(8) \quad \beta = B.$$

There is no evidence provided for the validity of this assumption. They have thus assumed that the probability,  $F(x)$ , that  $t$ , the number of items demanded during a delivery time, is less than or equal to some number  $x$ , is equal to the probability that the number of demands, during a delivery time, say  $y$ , is less than or equal to a number  $z$ , where  $z = x/B$ . Mathematically, this may be expressed as:

$$(9) \quad F(x) = \int_0^x \frac{e^{-t/\beta} t^p}{\beta^{p+1} p!} dt = \int_0^z \frac{e^{-y} y^p}{p!} dy = F(z)$$

where  $y = t/\beta$  and  $z = x/\beta$ .

The "safety level" is found by setting  $F(z)$  equal to 90%. Although the claim is made that this gives a 90% protection against stockout, in fact it gives only at best a 90% protection against the number of demands during delivery time being less than or equal to the average number of demands represented by the reorder point.<sup>[6]</sup> It assumes that every demand is the same size. It fails to distinguish between the two demand patterns shown in Table 2 -- in fact, it assumes that all demands received by an installation are of the Pattern 1 type -- 1, 1, 1, 1, 1; 2.5, 2.5, 2.5, 2.5, 2.5; etc.

Table 2. Demand Patterns Considered Equivalent by Harbridge House Plan

<u>Demand</u>	<u>Items Demanded</u>	
	<u>Pattern 1</u>	<u>Pattern 2</u>
1	10	1
2	10	16
3	10	2
4	10	25
5	10	6
	$B = \frac{50}{5} = 10$	$B = \frac{50}{5} = 10$

If this were the only difficulty involved in this assumption, the plan might be tolerable. However, examination of  $F(z)$  reveals another implicit assumption made by the crucial one,  $\beta = B$ . The distribution of number of demands during a lead time is a one-parameter distribution whose variance is equal to its mean and may in fact be represented by a Poisson distribution. Specifically,

---

[6] Order & Shipping Time Quantity plus Safety Level.

$$(10) \quad F(z) = \int_0^z \frac{e^{-y} y^p}{p!} dy = \sum_{q=p+1}^{\infty} \frac{e^{-z} z^q}{q!} = 1 - G(p)$$

where the latter function ( $G(p)$ ) is clearly a cumulative Poisson with mean and variance equal to  $z$ . The reader will find that by setting:

$$(11) \quad p + 1 = \text{average number of demands during a delivery time}^{[7]}$$

$$\text{and} \quad G(p) = \sum_{q=0}^p \frac{e^{-z} z^q}{q!} = .1$$

he may find the mean of the Poisson distribution ( $z$ ) in a standard table of the cumulative Poisson which has these properties. He can, in this manner duplicate the Safety Level Factors<sup>[8]</sup> for integral (whole number) values of  $p$  without ever referring to the gamma distribution.<sup>[9]</sup> The Pearson "Tables of the Incomplete  $\Gamma$  Function" cited above are more convenient computationally; nevertheless the distribution is a corrupt one-parameter version of the accepted gamma distribution and the use of these tables with a formidable name does not change the picture.

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[7] Found, as in Table D of the Annex of the Draft Instruction section of the Harbridge House report, "Installation of EIP at Stations" by dividing annual demands by 12, 8 or 6 for 30-day, 45-day and 60-day Order and Shipping Time, respectively. These are known as OST factors.

[8] Table C of the Annex of the Draft Instruction section of the Harbridge House Report. Since this report has occasional page numbers and no section numbers, a more precise reference is difficult.

[9] For non-integral values of  $p$ ,  $z$  may be found by graphical interpolation, within the limits of accuracy required by the table.

In either case, once  $z$  is found, the Safety Level Factor may be found in the manner used by Harbridge House.

Let  $A$  = annual items demanded

$t$  = months in order and ship time

$n$  = average number of demands in an order and ship time

$B$  = average demand size

$u$  = average number of items demanded per month.

Then:

$$(12) \quad \begin{aligned} \text{Safety Level Quantity} + \text{Order and Ship Time Quantity} &= zB, \\ \text{Order and Ship time Quantity} &= \frac{At}{12} = ut = nB \end{aligned}$$

and:

$$\begin{aligned} \text{Safety Level Quantity} &= zB - \frac{At}{12} \\ (13) \quad &= z\left(\frac{At}{12n}\right) - \frac{At}{12} \\ &= A\left(\frac{t}{12}\right) \left(\frac{z}{n} - 1\right) \\ &= \frac{A}{\text{Safety Level Factor}} \end{aligned}$$

Thus,

$$(14) \quad \text{Safety Level Factor} = \frac{12n}{t(z - n)}$$

and, incidentally,

$$(15) \quad \text{Order and Ship Time Factor} = \frac{12}{t} \quad .$$

These formulae will be found comparable to those of Harbridge House with  $u = z/\sqrt{n}$  used, rather than  $z$ , and the results are, of course, the same. [10]

It may appear that the question of whether or not the function actually used is a cumulative Poisson rather than gamma distribution is academic. Apparently Harbridge House investigators did not think so, and emphasized the need for a gamma distribution although, in fact they used a distribution with Poisson properties. The safety levels used fail to differentiate between an item that is demanded once a month "like clock work" and an item which has 12 demands in one month and none for the rest of the year. This would result in excessive safety levels on some items and practically no protection on others.

#### IV. Recommendations.

A first step forward would be to use the bona-fide gamma distribution. This would require estimates of both average demand and variance of demand. But, if the Department of Defense Instruction [11] is to be followed, taking into account both size of demand and variability of demand, this additional calculation is necessary. Some compensating simplification can be attained by substituting a "reorder point" for the cumbersome process of having an order and ship time quantity.

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[10] It should be obvious that a simplification could be brought about by eliminating the fiction of a separate Safety Level and Order and Ship Time Quantity. A "Reorder Point" =  $zB$  computed by the use of a "Reorder Point Factor" =  $12n/zt$  would simplify matters considerably for the "man on the Job" -- but this is a relatively minor criticism.

[11] DOD Instruction 4140.11, 24 June 1958.

This could be done as follows:

Let  $X_i$  be the amount demanded in the  $i^{\text{th}}$  month,  
 $i = 1, \dots, n$

$$\text{Then } \bar{X} = \frac{\sum x_i}{n}$$

$$\sigma_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}$$

For a one month order and ship time,  $\beta$  and  $p$  can be very simply calculated:

$$\beta = \frac{\sigma_x^2}{\bar{x}}$$

$$p = \frac{\bar{X}}{\beta} - 1$$

For an order and ship time of  $t$  months, let  $y_i$  be the amount demanded in the  $j^{\text{th}}$  order and ship time period. We can then estimate  $\bar{y}$  and  $\sigma_y^2$  by the following formulae:

$$\bar{y} = t \bar{x}$$

$$\sigma_y^2 = t \sigma_x^2$$

and, from this,

$$\beta = \frac{\sigma_y^2}{\bar{y}}$$

$$p = \frac{\bar{y}}{\beta} - 1.$$

An alternative procedure is to adopt the stuttering Poisson distribution described by Herbert Galliher.<sup>[12]</sup> This distribution is derived by assuming that the intervals between demands are randomly distributed and that the demand size is geometrically distributed. This seems to be a more satisfying characterization of the system and the average demand size  $B$  is a true parameter of the system (rather than an arbitrarily assumed parameter) thus possessing the simplification of the Harbridge House method without the artificiality.

The mean and variance,  $\sigma_y$ , of the stuttering Poisson during lead time has been found to be:

$$\bar{y} = ut$$

and

$$\sigma_y = \sqrt{2B - 1} \sqrt{ut}$$

where  $\bar{y}$  is the average use during lead time,  $u$  is the average monthly demand and  $t$  the number of months during lead time. The stuttering Poisson, for sufficiently large  $B$  is approximately normal with parameters as shown. The gamma assumption

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[12]"Interim Technical Report No. 9", MIT Project Fundamental Investigations in Methods of Operations Research under Contract DA-19-020-ORD-2684, Ordnance R&D Proj. No. TB-001, August 1958. Also see "Interim Technical Report No. 7", July 1957, and "Final Report of MIT Supply Control Procedure and Recommendations on Implementation", August 1960.

adopted by Harbridge House gave an identical mean, but variance, in the same terms as above as:

$$\sigma_y = \sqrt{B} \sqrt{ut} .$$

Thus the Harbridge House assumption gives a lower variance than the more reasonable assumptions about distribution and becomes progressively lower as B grows large. A recent informal communication from the Director of the "MIT Inventory Research Implementation Committee", OOR, indicates that at national level, even the stuttering Poisson is underestimating the variance. The Harbridge House assumptions at NACP's would thus be likely to grossly exaggerate the amount of protection provided by a given safety level.

## MEASURING SUPPLY EFFECTIVENESS

### I. Computation of Expected Value of Number of Stockouts During Lead Time.

Assume that  $F(x)$ , the probability that demand will not exceed the reorder point,  $x$ , in units may be expressed:

$$(16) \quad F(x) = \int_0^z \frac{e^{-y} y^p}{p!} dy$$

where  $z = x/B$  (the reorder point expressed in terms of number of demands of size  $B$ ) and  $y = t/B$  (the number of demands of size  $B$  occurring during lead time), and  $p + 1 =$  average number of demands during lead time. [13] The expected number of demands occurring against zero stock during lead time is the expected values of  $s$ , the number of units stockout where:

$$s = 0 \quad y < z$$

$$s = y - z \quad y \geq z.$$

Using  $E(x)$  to represent expected value of  $x$ ,

$$E(s) = \int_0^{\infty} s \frac{e^{-y} y^p}{p!} dy = \int_z^{\infty} (y - z) \frac{e^{-y} y^p}{p!} dy$$

As this is not a convenient form for tabulating purposes, we may rewrite it as

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[13] See Page 10 of this report.

$$\begin{aligned}
E(s) &= \int_0^{\infty} y \frac{e^{-y} y^p}{p!} dy - z \int_0^{\infty} \frac{e^{-y} y^p}{p!} dy \\
&= (p+1) \int_0^{\infty} \frac{e^{-y} y^{p+1}}{p!} dy - z \int_0^{\infty} \frac{e^{-y} y^p}{p!} dy
\end{aligned}$$

In this form the expected number of stockouts can be computed for given  $z$  from tables of Pearson (op. cit.), or, for integral values of  $p$ ,

$$E(s) = (p+1) \sum_{q=0}^{p+1} \frac{e^{-z} z^q}{q!} - z \sum_{q=0}^p \frac{e^{-z} z^q}{q!}$$

can be computed from a Poisson table.

## II. Computation of Supply Effectiveness.

Let

$N$  = number of demands occurring annually

$P$  = number of months in procurement cycle

Then the expected number of demands against zero stock during one year is equal to the procurement frequency times the  $E(s)$  or

$$\frac{E(s) \times 12}{P}$$

The proportion of demands during the year which occur at zero stock is then:

$$\frac{E(s) \times 12}{P \times N}$$

and

$$\% \text{ Supply effectiveness} = 100 \left[ 1 - \frac{12 \times E(s)}{N \times P} \right]$$

Of course,  $E(s)$ ,  $N$  and  $P$  will not be known precisely and, taking the risk of introducing bias for the sake of computational ease, we will assume that:

$$\text{Estimated \% Supply effectiveness} = 100 \left[ 1 - \frac{12 \times \hat{E}(s)}{\hat{N} \times \hat{P}} \right]$$

where  $\hat{x}$  is the estimate of  $x$ .

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